

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

## QUESTION NO. 1

- 1  $\frac{d}{dx} (\cos^{-1} \frac{x}{a}) = \dots\dots\dots$   
 (A)  $\frac{1}{1-x^2}$  (B)  $\frac{1}{1+x^2}$  (C)  $\frac{-1}{\sqrt{a^2-x^2}}$  (D)  $\frac{1}{\sqrt{a^2-x^2}}$
- 2 If  $y = \ln(\sin x)$ , then  $\frac{dy}{dx}$  is  
 (A)  $\tan x$  (B)  $\cot x$  (C)  $-\tan x$  (D)  $-\cot x$
- 3 The minimum value of the function  $f(x) = x^2 + 2x - 3$  is at  $x = \dots\dots\dots$   
 (A)  $-3$  (B)  $1$  (C)  $0$  (D)  $-1$
- 4  $\int x^{-1} dx = \dots\dots\dots$   
 (A)  $0 + c$  (B)  $-x^{-2} + c$  (C)  $\frac{x^{-2}}{-2} + c$  (D)  $\ln x + c$
- 5  $\int \frac{1}{1+\cos x} dx =$   
 (A)  $\frac{1}{2} \tan \frac{x}{2}$  (B)  $\tan \frac{x}{2}$  (C)  $\cot \frac{x}{2}$  (D)  $\frac{1}{2} \cot \left(\frac{x}{2}\right)$
- 6  $\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1-x^2}} =$   
 (A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{3}$  (C)  $\frac{\pi}{4}$  (D)  $\frac{\pi}{12}$
- 7 The order of the differential equation  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 3x = 0$  is  
 (A) 1 (B) 2 (C) 0 (D) 3
- 8 The solution set of inequality  $2x - 3 \geq 0$  is  
 (A)  $\left[\frac{3}{2}, \infty\right]$  (B)  $\left[\frac{2}{3}, \infty\right[$  (C)  $\left[\frac{2}{3}, \infty\right]$  (D)  $\left[\frac{3}{2}, 0\right[$
- 9 Perpendicular distance of the point  $P(6, -1)$  from the line  $3x + 4y + 1 = 0$  is  
 (A) 3 (B) 11 (C) 2 (D) 4
- 10 The coordinates of the point that divides the join of  $A(-6, 3)$  and  $B(5, -3)$  in the ratio 2 : 3 externally  
 (A)  $\left(-\frac{8}{3}, 1\right)$  (B)  $\left(\frac{8}{5}, -1\right)$  (C)  $(-28, 13)$  (D)  $(28, -13)$
- 11 If coordinates of the mid points of the sides of a triangle are  $(3, 2)$ ,  $(2, 3)$  and  $(1, -1)$ , then the area of the triangle is  
 (A) 10 sq. units (B) 6 sq. units (C) 11 sq. units (D) 5 sq. units
- 12 The latus rectum of a parabola  $y^2 = 4ax$  is  
 (A)  $y = -a$  (B)  $x = -a$  (C)  $y = a$  (D)  $x = a$
- 13 Condition that line  $y = mx + c$  is tangent to the circle  $x^2 + y^2 = a^2$  is  
 (A)  $c = \pm m \sqrt{1+a^2}$  (B)  $c = \pm m \sqrt{1-a^2}$  (C)  $c = \pm a \sqrt{1-m^2}$  (D)  $c = \pm a \sqrt{1+m^2}$
- 14 The projection of  $\underline{u} = a\underline{i} + b\underline{j} + c\underline{k}$  along  $\underline{i}$  is  
 (A) 0 (B) b (C) a (D) c
- 15 A constant force  $\underline{F}$  acting on a body, displaces it from A to B. The work done by  $\underline{F}$  is  
 (A)  $\underline{F} \cdot \underline{AB}$  (B)  $\underline{F} \times \underline{AB}$  (C)  $-\underline{F} \times \underline{AB}$  (D)  $-\underline{F} \cdot \underline{AB}$
- 16 The angle between the vectors  $4\underline{i} + 2\underline{j} - \underline{k}$  and  $-\underline{i} + \underline{j} - 2\underline{k}$  is  
 (A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{2}$  (D)  $\pi$
- 17 The coordinates of vertices of hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is  
 (A)  $(\pm a, 0)$  (B)  $(0, \pm b)$  (C)  $(0, \pm a)$  (D)  $(\pm b, 0)$
- 18 If  $f(x) = -2x+6$ , then  $f^{-1}(x) = \dots\dots\dots$   
 (A)  $6-2x$  (B)  $\frac{6-x}{2}$  (C)  $\frac{2}{6-x}$  (D)  $2x-6$
- 19  $\lim_{x \rightarrow 0} (1+3x)^{2/x} = \dots\dots\dots$   
 (A)  $e^2$  (B)  $e^8$  (C)  $e^6$  (D)  $e^4$
- 20 If  $f(x) = \tan x$ , then  $f^{-1}\left(\frac{\pi}{4}\right) = \dots\dots\dots$   
 (A) 1 (B)  $\frac{1}{2}$  (C) 2 (D)  $\frac{1}{3}$

**QUESTION NO. 2 Write short answers any Eight (8) of the following**

16

|    |   |
|----|---|
| 1  | Find the Domain and Range of $f(x) = x$   |
| 2  | Determine whether the function $f(x) = \frac{3x}{x^2+1}$ is even or odd                                     |
| 3  | For the functions $f(x) = 3x^4 - 2x^2$ , $g(x) = \frac{2}{\sqrt{x}}$ find $f \circ g(x)$ and $g \circ f(x)$ |
| 4  | Evaluate $\lim_{x \rightarrow \infty} \frac{5x^4 - 10x^2 + 1}{3x^3 + 10x^2 + 50}$                           |
| 5  | Find by definition the derivative of $\frac{1}{x^3}$  |
| 6  | Differentiate $(\sqrt{x} - \frac{1}{\sqrt{x}})^2$ w.r.t $x$   |
| 7  | Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$  |
| 8  | Differentiate $\sin x$ w.r.t $\cot x$   |
| 9  | For $f(x) = \ln \sqrt{e^{2x} + e^{-2x}}$ ; find $f'(x)$   |
| 10 | Find $y_1$ if $x^3 - y^3 = a^3$   |
| 11 | Find extreme values of $f(x) = 2x^3 - 2x^2 - 36x + 3$   |
| 12 | Find $\frac{dy}{dx}$ if $y = \ln(\tan h x)$   |

**QUESTION NO. 3 Write short answers any Eight (8) of the following**

16

|    |  |
|----|--|
| 1  | Find $dy$ if $y = x^2 + 2x$ , when $x$ changes from 2 to 1.8                                     |
| 2  | Evaluate $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$ ( $x > 0$ )                                   |
| 3  | Evaluate $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$  |
| 4  | Evaluate $\int e^x (\cos x + \sin x) dx$   |
| 5  | Evaluate $\int_1^2 \frac{x}{x^2+2} dx$   |
| 6  | Evaluate $\int_0^{\pi/3} \cos^2 x \cdot \sin x dx$   |
| 7  | Find the area between the x-axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$             |
| 8  | Solve the differential equation $\frac{dy}{dx} = -y$   |
| 9  | Show that the points $A(0,2)$ , $B(\sqrt{3}, -1)$ and $C(0,-2)$ are vertices of a right triangle |
| 10 | Find an equation of the line through $(-4, -6)$ and perpendicular to a line having slope $-3/2$  |
| 11 | Find whether the point $(5,8)$ lies above or below the line $2x - 3y + 6 = 0$                    |
| 12 | Find the lines represented by $20x^2 + 17xy - 24y^2 = 0$   |

**QUESTION NO. 4 Write short answers any Nine (9) of the following**

18

|    |   |
|----|---|
| 1  | Graph the solution set of $2x + y \leq 6$   |
| 2  | Find equation of circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$  |
| 3  | Find centre and radius of circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$   |
| 4  | Find vertex and directrix of parabola $x^2 = -16y$  |
| 5  | Find an equation of parabola whose focus is $F(-3,4)$ and directrix $3x - 4y + 5 = 0$   |
| 6  | Find foci and vertices of Hyperbola $\frac{y^2}{16} - \frac{x^2}{49} = 1$   |
| 7  | Find centre and eccentricity of $\frac{x^2}{4} - \frac{y^2}{9} = 1$   |
| 8  | Find magnitude of vector $\underline{u} = \underline{i} + \underline{j}$  |
| 9  | Find a unit vector in the direction of $\underline{v} = [-2, 4]$  |
| 10 | Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$   |
| 11 | If $\underline{v}$ is a vector for which $\underline{v} \cdot \underline{i} = 0$ , $\underline{v} \cdot \underline{j} = 0$ , $\underline{v} \cdot \underline{k} = 0$ Find $\underline{v}$ |
| 12 | Compute $\underline{a} \times \underline{b}$ if $\underline{a} = -4\underline{i} + \underline{j} - 2\underline{k}$ , $\underline{b} = 2\underline{i} + \underline{j} + \underline{k}$     |
| 13 | Find the value of $3\underline{j} \cdot \underline{k} \times \underline{i}$   |

Note: Attempt any Three questions from this section

10 x 3 = 30

|         |   |
|---------|---|
| Q.5-(A) | <p>If <math>f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, &amp; x \neq 2 \\ k, &amp; x = 2 \end{cases}</math></p> <p>Find k so that f(x) is continuous at x = 2</p>  |
| (B)     | <p>Prove that <math>y \frac{dy}{dx} + x = 0</math> if <math>x = \frac{1-t^2}{1+t^2}</math>, <math>y = \frac{2t}{1+t^2}</math></p>   |
| Q.6-(A) | <p>Evaluate <math>\int \frac{x \sin^{-1}x}{\sqrt{1-x^2}} dx</math></p>  |
| (B)     | <p>One vertex of a parallelogram is (1, 4), the diagonals intersect at (2, 1) and the sides have slopes 1 and <math>-\frac{1}{7}</math>. Find the other three vertices</p>        |
| Q.7-(A) | <p>Solve the differential equation <math>\sec^2x \tan y dx + \sec^2y \tan x dy = 0</math></p>   |
| (B)     | <p>Maximize <math>f(x,y) = x + 3y</math> subject to constraints<br/> <math>2x + 5y \leq 30</math>, <math>5x + 4y \leq 20</math>, <math>x \geq 0</math>, <math>y \geq 0</math></p> |
| Q.8-(A) | <p>Find equation of circle passing through A(-7, 7), B(5, -1), C(10, 0)</p>   |
| (B)     | <p>Show that mid-point of hypotenuse of a right angle triangle is equidistance from its vertices</p>  |
| Q.9-(A) | <p>If <math>y = a \cos(\ln x) + b \sin(\ln x)</math>, Prove that <math>x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0</math></p>   |
| (B)     | <p>Find the centre, foci, eccentricity and vertices of <math>9x^2 - 12x - y^2 - 2y + 2 = 0</math></p>   |

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

**QUESTION NO. 1**

- 1  $\int \tan x \, dx = \dots\dots\dots$   
 (A)  $\ln |\sec x| + c$  (B)  $\ln |\csc x| + c$  (C)  $\ln |\cos x| + c$  (D)  $\ln |\sin x| + c$
- 2  $\int \ln x^{-1} \, dx = \dots\dots\dots$   
 (A)  $x \ln x + x + c$  (B)  $-x \ln x + x + c$  (C)  $-x - x \ln x + c$  (D)  $x + \ln x + c$
- 3  $\int_0^{\pi/2} \sin x \, dx = \dots\dots\dots$   
 (A) 1 (B) 2 (C) 4 (D) 6
- 4  $\int \left(\frac{1}{x} + \ln x\right) e^x \, dx = \dots\dots\dots$   
 (A)  $\frac{1}{x} e^x + c$  (B)  $e^x \ln x + c$  (C)  $e^x \frac{\ln x}{x} + c$  (D)  $\frac{\ln x}{x} + c$
- 5 If  $m_1$  and  $m_2$  are slopes of two lines, then lines are perpendicular if  
 (A)  $m_1 m_2 = 1$  (B)  $m_1 = m_2$  (C)  $m_1 m_2 = -1$  (D)  $m_1 = -m_2$
- 6 An equation of horizontal line through point P(7, -9) is  
 (A)  $y = -9$  (B)  $y = 9$  (C)  $x = 7$  (D)  $x = -7$
- 7 The perpendicular distance of the line  $3x + 4y + 10 = 0$  from (0, 0) is  
 (A) 0 (B) 1 (C) 2 (D) 10
- 8  $x = 5$  is the solution of inequality  
 (A)  $2x - 3 > 0$  (B)  $2x + 3 < 0$  (C)  $x + 4 < 0$  (D)  $x < 0$
- 9 The radius of circle  $(x - 5)^2 + (y - 3)^2 = 8$  is  
 (A) 2 (B)  $2\sqrt{2}$  (C) 4 (D) 64
- 10 The vertex of parabola  $(x - 1)^2 = 8(y + 2)$  is  
 (A) (1, 2) (B) (0, 1) (C) (-1, -2) (D) (1, -2)
- 11  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is the standard equation of  
 (A) Circle (B) Parabola (C) Ellipse (D) Hyperbola
- 12 If  $\underline{u} = 2\underline{i} + 4\underline{j} + 7\underline{k}$  and  $\underline{v} = 2\underline{i} + 6\underline{j} + \alpha \underline{k}$  are perpendicular, then  $\alpha = ?$   
 (A) -4 (B) 4 (C) 28 (D) 0
- 13  $2 \underline{k} \cdot \underline{j} \times \underline{i}$  is equal to  
 (A) 1 (B) -1 (C) -2 (D) 2
- 14 If  $\underline{u} = 2\underline{i} - \underline{j} - 2\underline{k}$ , then  $|\underline{u}| = ?$   
 (A) 2 (B) 3 (C) 4 (D) 5
- 15  $f(x) = \cos x + \sin x$  is ..... function  
 (A) Even (B) Odd (C) Both even and odd (D) Neither even nor odd
- 16  $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{n}\right)^{2n} = \dots\dots\dots$   
 (A)  $e^2$  (B)  $e^4$  (C)  $e^6$  (D)  $e^9$
- 17  $\frac{d}{dx} \left(\frac{1}{\sqrt{x}}\right) = \dots\dots\dots$   
 (A)  $\frac{1}{2x\sqrt{x}}$  (B)  $-\frac{1}{2x\sqrt{x}}$  (C)  $\frac{\sqrt{x}}{2}$  (D)  $-\frac{\sqrt{x}}{2}$
- 18  $\frac{d}{dx} (\cos x^2) = \dots\dots\dots$   
 (A)  $-\sin x^2$  (B)  $2x \sin x^2$  (C)  $-2x \sin x^2$  (D)  $\sin x \cdot 2x$
- 19 If  $y = 5 e^{3x-4}$ , then  $\frac{dy}{dx} = \dots\dots\dots$   
 (A)  $15 e^{3x-4}$  (B)  $-15 e^{3x-4}$  (C)  $20 e^{3x-4}$  (D)  $-20 e^{3x-4}$
- 20 If  $y = \sin 3x$ , then  $y_2 = \dots\dots\dots$   
 (A)  $3 \cos 3x$  (B)  $9 \sin 3x$  (C)  $9 \cos 3x$  (D)  $-9 \sin 3x$



QUESTION NO. 2 Write short answers any Eight (8) of the following

16

|    |   |
|----|---|
| 1  | Find $\text{fof}(x)$ for $f(x) = \sqrt{x+1}$ , $g(x) = \frac{1}{x^2}$ , $x \neq 0$    |
| 2  | Find $f^{-1}(x)$ if $f(x) = (-x+9)^3$   |
| 3  | Find $f(x-1)$ if $f(x) = \sqrt{x+4}$  |
| 4  | Find $\frac{f(a+h)-f(a)}{h}$ , for $f(x) = \sin x$                                    |
| 5  | If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ , show that $\frac{2x dy}{dx} + y = 2\sqrt{x}$ |
| 6  | Differentiate w.r.t x If $y = \frac{2x-1}{\sqrt{x^2+1}}$                              |
| 7  | Differentiate $\frac{x^2+1}{x^2-1}$ w.r.t, $x^3$                                      |
| 8  | Find $\frac{dy}{dx}$ if $y = x \cos y$  |
| 9  | Find $\frac{dy}{dx}$ if $y = e^{-x} (x^3+2x^2+1)$                                     |
| 10 | Find $\frac{dy}{dx}$ if $y = \ln(\tan h x)$   |
| 11 | Find $\frac{dy}{dx}$ if $y = \sin h^{-1}(x^3)$  |
| 12 | Find $y_2$ if $y = x^2 e^{-x}$  |

QUESTION NO. 3 Write short answers any Eight (8) of the following

16

|    |  |
|----|--|
| 1  | Find $dy$ if $y = x^2$ and $x$ changes from 2 to 2.01  |
| 2  | Evaluate $\int \frac{\sin x + \cos^3 x}{\cos^2 x \sin x} dx$   |
| 3  | Evaluate the given integral $\int \sin^2 x dx$   |
| 4  | Evaluate $\int \cos x \left( \frac{\ln \sin x}{\sin x} \right) dx$   |
| 5  | Find the antiderivative of $\sin^{-1} x$   |
| 6  | Evaluate the definite integral $\int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta$  |
| 7  | Solve the differential equation $\frac{dy}{dx} = y^2 + 1/e^{-x}$   |
| 8  | The length of perpendicular from the origin to a line is 5 units and the inclination of this perpendicular is $120^\circ$ . Find the slope of the line |
| 9  | Find an equation of the line through $(-5, -3)$ and $(9, -1)$  |
| 10 | Convert the given equation into normal form : $4x + 7y - 2 = 0$  |
| 11 | Find an equation of each of the lines represented by : $20x^2 + 17xy - 24y^2 = 0$  |
| 12 | Find the interior angles (any two) of the triangle whose vertices are :<br>$A(6, 1)$ , $B(2, 7)$ , $C(-6, -7)$   |

QUESTION NO. 4 Write short answers any Nine (9) of the following

18

|    |   |
|----|---|
| 1  | Graph the solution set of $5x - 4y \leq 20$   |
| 2  | Find the sum of $\vec{AB}$ and $\vec{CD}$ given the four points $A(1, -1)$ , $B(2, 0)$ , $C(-1, 3)$ and $D(-2, 2)$  |
| 3  | Find $2\vec{CB} - 2\vec{CA}$ if $A = (2, 5)$ , $B = (-1, 1)$ and $C = (2, -6)$  |
| 4  | Find a vector whose magnitude is 2 and is parallel to $-\underline{i} + \underline{j} + \underline{k}$  |
| 5  | If $\underline{v}$ is a vector for which $\underline{v} \cdot \underline{i} = 0$ , $\underline{v} \cdot \underline{j} = 0$ , $\underline{v} \cdot \underline{k} = 0$ then find $\underline{v}$  |
| 6  | A force $\vec{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$ is applied at $P(1, -2, 3)$ . Find its moment about the point $Q(2, 1, 1)$   |
| 7  | If $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$ , $\underline{b} = \underline{i} - \underline{j} + \underline{k}$ find $\underline{b} \times \underline{a}$ and show $\underline{b} \times \underline{a}$ is perpendicular to $\underline{a}$ |
| 8  | Find centre and radius of circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$  |
| 9  | Find the length of the tangent from the point $P(-5, 10)$ to the circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$  |
| 10 | Find focus and vertex of the parabola $x^2 = -16y$  |
| 11 | Find eccentricity and vertices of $9x^2 - 12x - y^2 - 2y + 2 = 0$   |
| 12 | Find an equation of the tangent to the conic $x^2 - xy + y^2 - 2 = 0$ at the point whose ordinate is $\sqrt{2}$   |
| 13 | Find the volume of tetrahedron with the vertices $A(2, 1, 8)$ , $B(3, 2, 0)$ , $C(2, 1, 4)$ and $D(3, 2, 10)$   |

D4K-II-21  
SECTION-II

Note: Attempt any Three questions from this section

10 x 3 = 30

|          |  |
|----------|--|
| Q.5- (A) | Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$   |
| (B)      | Find $\frac{dy}{dx}$ if $x = a(\cos t + \sin t)$ , $y = a(\sin t - t \cos t)$  |
| Q.6- (A) | Evaluate $\int \sqrt{x^2 + 4} \, dx$   |
| (B)      | One vertex of a parallelogram is (1 , 4) , the diagonals intersect at (2 , 1) and the sides have slope 1 and $-\frac{1}{7}$ . Find the other three vertices      |
| Q.7-(A)  | Evaluate $\int_0^{\pi/4} \frac{\cos\theta + \sin\theta}{\cos 2\theta + 1} \, d\theta$  |
| (B)      | Graph the feasible region of the following system of linear inequalities and find the corner points $3x + 7y \leq 21$ , $x - y \leq 3$ , $x \geq 0$ , $y \geq 0$ |
| Q.8-(A)  | Find the coordinates of the points of intersection of the line $x + 2y = 6$ with the circle $x^2 + y^2 - 2x - 2y - 39 = 0$                                       |
| (B)      | Use vectors prove that $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$  |
| Q.9-(A)  | If $y = (\cos^{-1}x)^2$ then prove that $(1 - x^2) y_2 - xy_1 - 2 = 0$   |
| (B)      | Find an equation of the parabola whose focus is (-3 , 4) and directrix is $3x - 4y + 5 = 0$  |