

Time allowed: 3 Hrs

Mathematics (Part - I)

Marks: 100

Fresh / Reappear

Code - A

Note: There are three sections of the paper, A, B & C. Attempt Section - A on the same paper and return it to the superintendent within the given time. Mobile phone etc. are not allowed in the examination hall.

Time: 20 Mins

Section "A"

Marks: 20

Q.1 Write the correct option i.e. A, B, C or D in the empty box provided opposite to each part. No marks will be awarded for cutting, erasing or over writing.

- i. If $Z + \bar{Z} = 0$ then Z is A
 A. Real B. Imaginary C. 1 D. 9
- ii. The $x + iy$ form of $(1-3i)^{-1}$ is A
 A. $\frac{1}{10} + \frac{3}{10}i$ B. $-\frac{1}{10} - \frac{3}{10}i$ C. $\frac{3}{10} - \frac{3}{10}i$ D. $\frac{1}{3} + \frac{3}{5}i$
- iii. ${}^n P_4 = 6 ({}^n P_3)$, then $n =$ C
 A. 14 B. 1 C. 9 D. 11
- iv. $n! > n^2$ holds if D
 A. $n \in \mathbb{N}$ B. $n \geq 2$ C. $n \geq 3$ D. $n \geq 4$
- v. $\sum_{k=1}^n i^k =$ B
 A. 0 B. 1 C. -1 D. k^2
- vi. If $y = \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \dots$ where $0 < x < 3$ then $x =$ A
 A. $\frac{3y}{1+y}$ B. $\frac{1+y}{3y}$ C. y^2 D. $3y^n$
- vii. If $\left| \begin{matrix} x-2 & 1 \\ 5 & x+2 \end{matrix} \right| = 0$ then $x =$ D
 A. 0 B. 3 C. -3 D. ± 3
- viii. Geometric Means between the numbers $\sqrt{5} - 4$ and $\sqrt{5} + 4$ is C
 A. 4 B. -1 C. Does not exist D. ± 3
- ix. The general term of the sequence $-1, 1, -1, 1, \dots$ B
 A. $(-1)^{n+1}$ B. $(-1)^n$ C. $(-1)^{2n}$ D. $-n$
- x. ${}^n C_3 + {}^n C_7 =$ A
 A. ${}^n C_4$ B. ${}^n C_7$ C. ${}^n C_1$ D. 1
- xi. Solution of the inequality $x - 5 > 0$ is C
 A. R B. $x \geq 5$ C. $x > 5$ D. $x < 5$
- xii. $2 \sin^2(\beta/2) =$ B
 A. $\cos \beta$ B. $1 - \cos \beta$ C. $1 + \cos \beta$ D. $\cos^2 \beta/2$
- xiii. The graph of an odd function is symmetric about C
 A. X-axis B. Y-axis C. Origin D. Line $y = x$
- xiv. If θ is in 3rd Quadrant then $\theta/2$ will be in B
 A. 1st Quadrant B. 2nd Quadrant C. 3rd Quadrant D. 4th Quadrant
- xv. If M_{ij} is a minor of an entry a_{ij} of a square matrix A, then the Cofactor $A_{ij} =$ C
 A. $(-1)^{i+j} M_{ij}$ B. $(-1)^{i-j} M_{ij}$ C. $(-1)^{+j} M_{ij}$ D. $(-1)^i M_{ij}$
- xvi. The value $r_1 r_2 + r_2 r_3 + r_3 r_1 =$ B
 A. R^2 B. S^2 C. Δ^2 D. Does not exist
- xvii. $\sin^{-1}(1/2) =$ C
 A. π B. $\pi/3$ C. $\pi/6$ D. $\frac{\pi}{4}$
- xviii. Which one of the following function is Odd. C
 A. $y = x^2 + 2$ B. $y = 2x + 6x^3$ C. $y = \sin x$ D. $y = (x+1)^2$
- xix. $\frac{b^2 - a^2 - c^2}{2ac} =$ D
 A. $\cos \beta$ B. $\cos \alpha$ C. $\cos \gamma$ D. $-\cos \beta$
- xx. $\sum_{k=1}^n k^3 =$ C
 A. $\frac{n(n+1)}{2}$ B. $\frac{n(n+1)^2}{4}$ C. $\left[\frac{n(n+1)}{2} \right]^2$ D. $\frac{n(n+1)(2n+1)}{6}$

Section "B"

Marks: 50

Q.2 Attempt any TEN parts. All parts carry equal marks.

- i. Separate real and imaginary parts of complex number $\left(\frac{5-2i}{2+3i}\right)^{-1}$.
- ii. Solve for x , $\begin{vmatrix} -1 & 0 & 1 \\ x^2 & 1 & x \\ 2 & 3 & 4 \end{vmatrix} = -6$
- iii. Find the projection of the vector $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ on the vector $\mathbf{b} = 4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$.
- iv. Find the sum of all the numbers divisible by 5 from 25 through 350.
- v. Sum the series. $1.5.9 + 2.6.10 + 3.7.11 + \dots$ to n terms.
- vi. Find the value of n . $\frac{n(n!)}{(n-5)!} = \frac{12(n!)}{(n-4)!}$
- vii. If x is nearly equal to unity then show that $px^p - qx^q = (p-q)x^{p+q}$
- viii. If two dice are rolled. Find the probability of obtaining a total of 7 or 11.
- ix. Prove that $\cos 3\alpha(1 - 2\sin \alpha) = \cos 3\alpha - (\sin 4\alpha - \sin 2\alpha)$
- x. What is the vertex angle of an isosceles triangle whose equal sides are 13 ft long. If the area is 50 ft^2 .
- xi. Write $\cos(\sin^{-1}x)$ as an algebraic expression.
- xii. Minimize $f(x, y) = 3x + 4y$ subject to the constraints $2x + 3y \geq 6$, $x + y \leq 8$, $x \geq 0$, $y \geq 0$
- xii. Find the point of intersection of the functions graphically $f(x) = 3x - 2$, $g(x) = -x + 6$

Section "C"

Marks: 30

Note: Attempt any THREE questions. All questions carry equal marks.

Q.3 a. If x^4 and higher power are neglected then show that $(1+x)^{1/4} + (1-x)^{1/4} = a - bx^2$. Also find a and b .b. Find the value of λ if A is singular

$$A = \begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix}$$

Q.4 a. Determine whether the function. $f(x) = x\sqrt{x^2+3}$ is even, odd or neither.b. Show that $\tan^{-1}x + \tan^{-1}\frac{1}{x} = \frac{\pi}{2}$ Q.5 a. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ verify that $(A^{-1})^t = (A^t)^{-1}$ b. Solve the quadratic equation $2z^2 + 15 = 4z$ Q.6 a. For the vectors: $\mathbf{a} = 3\mathbf{i} + 2\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{c} = -\mathbf{j} + 4\mathbf{k}$ Verify that $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b}$ b. For what value of n , $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the Geometric Mean between a and b .where a and b are not zero simultaneously.