MATHEMATICS HSSC–II

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

SECTION – B (Marks 48)

Q. 2 Solve the following Questions. **(12 x 4 = 48)**

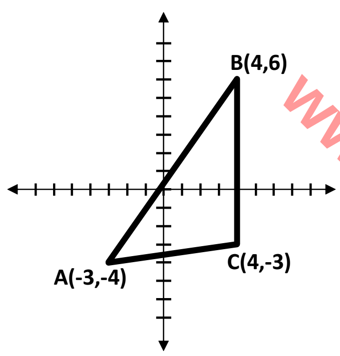
(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

(i)	If $f(x)=\frac{3x+2}{2x-1}$, find $f^{-1}(x)$ and also show that $f^{-1}(f(x))=x$	**04**	**OR**	Find $\frac{dy}{dx}$ if $2y^3-3xy^2+2x^2y+5x=6$ Also find the value of $\frac{dy}{dx}$ at $(1,1)$	**04**
(ii)	Find the derivative of $y=(2\sqrt{x}+2)(x-\sqrt{x})$	**04**	**OR**	Discuss the continuity of the function at $x=1$ $f(x)=\begin{cases}3x-1 & \text{if } x<1\\ 4 & \text{if } x=1\\ 2x & \text{if } x>1\end{cases}$	**04**
(iii)	If $y=Cot(qCot^{-1}x)$ then show that $(1+x^2)y_1-q(1+y^2)=0$	**04**	**OR**	Evaluate $\lim_{\theta\to 0}\frac{\text{Sec}\theta-1}{\theta\,\text{Sec}\theta}$	**04**
(iv)	Examine the function x^3-6x^2+9x+3 for extreme values.	**04**	**OR**	Use the differential to approximate value of $\text{Sin}\,61°$	**04**
(v)	Find the area bonded by the curve $y=x^3-9x$ and the x-axis.	**04**	**OR**	Find the area of the region bonded by $10x^2-xy-21y^2=0$ and $x+y+1=0$	**04**
(vi)	Solve the differential equation $\frac{dy}{dx}=\frac{3}{4}x^3+x-3$ if $y=0$, when $x=2$	**04**	**OR**	Find the point P on the joint of $A(1,4)$ and $B(5,6)$ that is twice as far from A as B is from A and lies on opposite side of A as B does.	**04**
(vii)	The length of perpendicular form origin to the line is 8 unit and angle of inclination is $30°$. Find the slope and y-intercept of the line.	**04**	**OR**	Integrate $\int\frac{x\,\text{Sin}^{-1}x}{\sqrt{1-x^2}}dx$	**04**
(viii)	Find the equation of a circle passing through the point $(-2,-5)$ and touching the line $3x+4y-24=0$ at the point $(4,3)$	**04**	**OR**	Graph the feasible region of the following system of linear inequalities by shading and find the corner points $3x+2y\ge 6,\quad x+y\le 4,\quad x\ge 0,\quad y\ge 0$	**04**
(ix)	Graph the feasible region of the following system of linear inequalities by shading and find the corner points. $5x+7y\le 35,\quad x-2y\le 4,\quad x\ge 0,\quad y\ge 0$	**04**	**OR**	Find the equation of parabola having Focus $(-3,4)$ and directrix $3x+2y-3=0$	**04**
(x)	Prove that altitudes of a triangle are concurrent (by vector method).	**04**	**OR**	Find the value of C, when the line $5x+2y+C=0$ will touch the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=1$	**04**
(xi)	Find the points of intersection to the two conics $\frac{x^2}{18}+\frac{y^2}{8}=1$ and $\frac{x^2}{3}-\frac{y^2}{3}=1$	**04**	**OR**	If $\bar{u}=2\hat{i}+3\hat{j}+4\hat{k}$, $\bar{v}=\hat{i}+4\hat{j}+3\hat{k}$ and $\bar{w}=\hat{i}+7\hat{j}+\lambda\hat{k}$ represent the side of a triangle. Find the value of λ	**04**
(xii)	Find the equation of an ellipse with foci $(\pm\sqrt{5},0)$ and passing through the point $\left(\frac{3}{2},\sqrt{3}\right)$	**04**	**OR**	Find the moment about $(1,1,1)$ of each of the concurrent forces $\hat{i}-2\hat{j}$, $3\hat{i}+2\hat{j}-\hat{k}$ and $5\hat{j}+2\hat{k}$ where $P(2,0,1)$ is the point of concurrency.	**04**

SECTION – C (Marks 32)

Note: Solve the following Questions. **(4 x 8 = 32)**

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

Q.3	Let $f(x)=\begin{cases} px+2 & \text{if } 0\le x<2 \\ 7-qx & \text{if } 2\le x<4 \\ 2x+1 & \text{if } 4\le x<6 \end{cases}$ Find the values of p and q such that $f(x)$ is continuous at $x=2$, and $x=4$, sketch the graph of $f(x)$ after finding the values of p and q.	**08**	**OR**	A box with a square base and open top is to have a volume 32 cubic dm. Find the dimensions of the box that will require the least material.	**08**
Q.4	Evaluate $\int \frac{2x^2-x-7}{(x+2)^2(x^2+2x+5)}dx$	**08**	**OR**	Find the equation of tangent of an ellipse $\frac{x^2}{128}+\frac{y^2}{18}=1$ which are parallel to the line $3x+8y+1=0$ Also find the point of contact.	**08**
Q.5	The diagram shows a triangle ΔABC where $A(-3,-4)$, $B(4,6)$ and $C(4,-3)$  a. Write the equation of sides $\overline{AB}$ and $\overline{AC}$. b. Find interior angle, $\angle A$. c. Find the area of ΔABC d. Find the perpendicular distance from point $C(4,-3)$ to line $\overline{AB}$	**08**	**OR**	A factory produces two items ceiling lights and ceiling fans by using two machines A and B. Machine A has at most 120 hours available and machine B has maximum 144 hours available. Manufacturing of a ceiling light requires 5 hours in machine A and 4 hours in machine B, while manufacturing of a ceiling fan requires 4 hours in machine A and 8 hours in machine B. If the factory gets a profit of Rs 50 per ceiling light and Rs 80 per celling fan, how many celling lights and ceiling fans should be manufactured to get maximum profit?	**08**
Q.6	Find the centre, foci, eccentricity, vertices and equation of directrices of the conic $9x^2-y^2-12x-2y+2=0$	**08**	**OR**	Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{Cos}^2\theta\, Cot^2\theta\, d\theta$	**08**

—— (B) ——

MATHEMATICS HSSC–II

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

SECTION – B (Marks 48)

Q. 2 Solve the following Questions. **(12 x 4 = 48)**

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

(i)	Given that $f(x)=x^3-px^2+qx+1$ if $f(3)=-5$ and $f(-2)=5$ then find the values of p and q	**04**	**OR**	Show that $\frac{dy}{dx}=\frac{y}{x}$ if $\frac{x}{y}=\tan^{-1}\left(\frac{x}{y}\right)$	**04**
(ii)	Find the derivative of $x^{\frac{3}{4}}$ by using definition w.r.t. x. Also calculate the derivative at $x=16$	**04**	**OR**	For the given function $f(x)=\begin{cases}2ax+3 & \text{if } x<2\\ 2x+2a & \text{if } x\geq 2\end{cases}$ Find the value of a that $\lim_{x\to 2} f(x)$ exists	**04**
(iii)	Evaluate $\int_0^{\frac{\pi}{2}} \text{Cos}^3 x\sqrt{\text{Sin}\,x}\,dx$	**04**	**OR**	Evaluate the limit. $\lim_{\theta\to 0}\frac{1-\text{Cos}\,3\theta}{1-\text{Cos}\,5\theta}$	**04**
(iv)	Find the point on the curve $y=x^2+2$ that is closest to the point $(18,2)$	**04**	**OR**	Evaluate $\int \ln\left(x+\sqrt{x^2+1}\right)dx$ (by parts)	**04**
(v)	Find $\frac{dy}{dx}$ if $y=\ln\left[\frac{x\left(x^2-3\right)^2}{\left(x^2-4\right)^{\frac{1}{2}}}\right]$	**04**	**OR**	Find the area above the x-axis bounded by the curve $y^2=3-x$ from $x=-1$ to $x=2$	**04**
(vi)	Find the equation of an ellipse with vertices $(-1,1),(5,1)$ and foci $(4,1),(0,1)$	**04**	**OR**	If two vertices of an equilateral triangle are $A(-4,0)$ and $B(4,0)$. Find the third vertex. How many of these triangles are possible?	**04**
(vii)	Graph the feasible region subject to the following constraints $2x+8y\leq 60,\ 4x+4y\leq 60,\ x\geq 0,\ y\geq 0$ Also find the corner points.	**04**	**OR**	Find a joint equation of the straight line through the origin represented by $2x^2-5xy+3y^2=0$	**04**
(viii)	Find the equation of a straight line through intersection of the lines $x+2y+3=0,\ 3x+4y+7=0$ and making equal intercept on axes.	**04**	**OR**	Graph the feasible region subject to the following constraints $3x-y\geq -1,\ x+y\leq 5,\ x\geq 0,\ y\geq 0$ Also find the corner points.	**04**
(ix)	In any triangle ABC, Prove that $c^2=a^2+b^2-2ab\,\text{Cos}\,C$ (Use vector method)	**04**	**OR**	Write the equation of parabola with given elements directrix $y=3$, vertex $(2,2)$	**04**
(x)	Solve the differential equation $y-x\frac{dy}{dx}=3\left(1+x\frac{dy}{dx}\right)$ Also find the particular solution if $y=7$ and $x=16$	**04**	**OR**	Find the equation of the normal to the parabola $y^2=8x$ which is parallel to the line $2x+3y=10$	**04**
(xi)	Find the equation of hyperbola with center $(2,2)$ horizontal transverse axis of length 6 and eccentricity $e=2$	**04**	**OR**	Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$. Also find the sine of the angle between them, $\underline{a}=2\hat{i}-3\hat{j}+4\hat{k},\ \ \underline{b}=-\hat{i}-\hat{j}-\hat{k}$	**04**
(xii)	Write the equation of tangent line to the circle $x^2+y^2+4x+2y=0$ drawn from $(-1,2)$. Also find the tangential distance.	**04**	**OR**	Find the volume of tetrahedron whose vertices are $A(2,1,8),\ B(3,2,9),\ C(2,1,4)$ and $D(3,3,10)$	**04**

SECTION – C (Marks 32)

Note: Solve the following Questions. **(4 x 8 = 32)**

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

Q.3	Let $f(x)=\begin{cases} lx+2m & \text{if } 0\le x<1 \\ mx+2 & \text{if } 1\le x<2 \\ 3lx+4 & \text{if } 2\le x<4 \end{cases}$	a. Find the values of l and m such that function f is continuous at $x=1$ and $x=2$ b. After finding values of l and m, sketch the graph of $f(x)$	**08**	**OR**	Discuss the function defined as $f(x)=\text{Sin}\,x+\frac{1}{2\sqrt{2}}\text{Cos}\,x$ for extreme values in the interval $(0,2\pi)$	**08**
Q.4	Find the centre, foci, eccentricity, vertices and directirices of the conic $9x^2-18x+4y^2+8y-23=0$		**08**	**OR**	Show that $\int e^{ax}\,\text{Sin}\,bx\,dx=\frac{1}{\sqrt{a^2+b^2}}\text{Sin}\left(bx-\tan^{-1}\left(\frac{b}{a}\right)\right)+c$ By using this formula evaluate $\int e^{2x}\,\text{Sin}\,3x\,dx$	**08**
Q.5	Find the distance between the parallel lines: $12x+5y-6=0,\ 12x+5y+13=0$ a. Sketch the lines. b. Find the equation of parallel lines lying mid way between them. c. Write the equation of line in two intercept from.		**08**	**OR**	A toy factory manufactures two types of toys: remote car and scooty by using two machines A and B. Machine A has at most 110 hours available and machine B has a maximum of 300 hours available. Manufacturing a remote car requires 5 hours in machine A and 10 hours in machine B. While manufacturing of scooty requires 30 hours in machine A and 20 hours in machine B. If the factory gets profit of Rs. 300 per remote car and Rs. 400 per scooty, how many both toys should be manufactured to get maximum profit?	**08**
Q.6	Evaluate $\int\frac{2x^2-x-7}{(x+3)(x-2)(x^2-2x+3)}dx$		**08**	**OR**	Find the equation of tangent and normal at a point $\left(5,\frac{16}{9}\right)$ to the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$. Also find the value of c, when the line $y=-x+c$ will touch the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$	**08**

—— (D) ——