# MATHEMATICS HSSC–II

36

## SECTION – A (Marks 20)

**Time allowed: 25 Minutes** | **Version Number** | 1 | 8 | 7 | 1 |

**Note:** **Section – A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.**

**Q. 1** **Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.**

1) Find the range of the function $y = f(x) = 2 + \sqrt{x-1}$

A. $2 \le y < \infty$ B. $3 \le y < \infty$ C. $1 \le y < \infty$ D. $-1 \le y < \infty$

2) _____ is the value of the $\lim_{x \to -\infty} \frac{|x|}{x}$

A. $-1$ B. $1$ C. $0$ D. $\infty$

3) $\frac{d}{dx}(\log_a x) = ?$

A. $\frac{\ln a}{x}$ B. $\frac{1}{x}$ C. $\frac{1}{x \ln a}$ D. $\frac{a}{x \ln a}$

4) $\frac{d}{dx}(e^x + e^{-x}) = ?$

A. $2\sinh x$ B. $2\cosh x$ C. $\sinh x$ D. $\cosh x$

5) What is $x$-coordinate of the point at which tangent to the graph of $y = x^2 - 2x$ is horizontal?

A. $-1$ B. $+1$ C. $+2$ D. $-2$

6) The anti-derivative of $\frac{2}{x^2+4}$ with respect to $x$ is:

A. $2\tan^{-1}(\frac{x}{2})$ B. $\tan^{-1}(\frac{x}{2})$ C. $\frac{1}{2}\tan^{-1}(\frac{x}{2})$ D. $2\tan^{-1}(x)$

7) The value of $\int_1^3 f(x)dx$ , if $\int_0^3 f(x)dx = 4$ and $\int_0^1 f(x)dx = -1$ is:

A. 3 B. 4 C. 5 D. 2

8) The value of $\int_0^{\frac{\pi}{2}} \cos x dx$ is:

A. $-1$ B. $+1$ C. $-2$ D. $0$

9) What is the area bounded by the line $y = x$, the x-axis and the lines $x = 0$ and $x = 1$ ?

A. 0 B. 1 C. 2 D. 0.5

10) The evaluation of $\int_0^1 (\sinh x + \cosh x)dx$ is:

A. $e+1$ B. $e-1$ C. $2e$ D. $e$

11) The centroid of the triangle with vertices $A(3,-5)$, $B(-7,4)$ and $C(10,-2)$ is:

A. $(2,1)$ B. $(-2,1)$ C. $(-2,-1)$ D. $(2,-1)$

12) What is the mid-point of BC if C is the mid-point of the join of $A(-1,4)$ and $B(7,12)$ ?

A. $(3,8)$ B. $(5,10)$ C. $(1,6)$ D. $(4,4)$

13) How many straight lines are represented by the equation $4x^2 + 24xy + 11y^2 = 0$ ?

A. 0 B. 1 C. 4 D. 2

14) In which quadrant does the solution region of the inequalities $x \le -1, y \ge 1$ lie?

A. Quadrant I B. Quadrant II C. Quadrant III D. Quadrant IV

15) What is the radius of the circle with equation $x^2 + y^2 + 2r\cos\theta x + 2r\sin\theta y = 0$ ?

A. $r$ B. 1 C. $r^2$ D. 0

16) Which of the following conditions is that the point $(b,c)$ lies outside the circle $x^2 + y^2 = a^2$ ?

A. $b^2 + c^2 > a^2$ B. $b^2 + c^2 < a^2$ C. $a^2 + b^2 < c^2$ D. $a^2 + b^2 > c^2$

17) Which of the following points does **NOT** lie on the parabola $y^2 = 4ax$ ?

A. $(2a,a)$ B. $(a,-2a)$ C. $(at^2,2at)$ D. $(at^2,-2at)$

18) The condition that the second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a hyperbola is:

A. $h^2 - ab < 0$ B. $h^2 - ab > 0$ C. $h^2 - ab = 0$ D. $h^2 = a + b$

19) Find the value of $\lambda$ for which the vector $2\underline{i} - \underline{j} + 2\underline{k}$ is perpendicular to the vector $3\underline{i} + 2\lambda\underline{j}$

A. $-2$ B. $-3$ C. 3 D. 6

20) What are the direction cosines of the vector $\sqrt{3}\underline{i} - \sqrt{3}\underline{j} + \sqrt{3}\underline{k}$ ?

A. $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ B. $1,1,-1$ C. $1,-1,1$ D. $\sqrt{3},\sqrt{3},\sqrt{3}$

# MATHEMATICS HSSC–II

**Time allowed: 2:35 Hours** **Total Marks Sections B and C: 80**

**NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.**

## SECTION – B (Marks 40)

**Q. 2 Attempt any TEN parts. All parts carry equal marks.** **( 10 x 4 = 40 )**

(i) If $f(x)=\begin{cases} x+2\ , & x\le -1 \\ c+2\ , & x>-1 \end{cases}$, find "$c$" so that $\lim_{x\to -1} f(x)$ exists.

(ii) Find $\frac{dy}{dx}$ using the first principle if $y=\frac{1}{\sqrt{x+a}}$

(iii) If $y=\tan(p\,\mathrm{Tan}^{-1}x)$ then show that $(1+x^2)y_1-p(1+y^2)=0$

(iv) Find the intervals in which the functions $f(x)=x^2+3x+2\ ;\ x\in(-4,1)$ is:

a. increasing b. decreasing

(v) Evaluate $\int\sqrt{1-\cos 2x}\ dx$ (where $1-\cos 2x>0$)

(vi) Evaluate $\int_{-1}^{2}(x+|x|)dx$

(vii) Solve the following differential equation: $xdy+y(x-1)dx=0$

(viii) The xy-coordinates are rotated about the origin through the angle of measure $\theta=30°$ and the new axes are OX and OY. Find the xy-coordinates of point P with the given XY-coordinates of point P as $(-5,3)$.

(ix) Find the angle from the line with slope $\frac{-7}{3}$ to the line with slope $\frac{5}{2}$.

(x) Graph the feasible region of the system of linear inequalities:

$3x+2y\ge 6$

$x+y\le 4$

$x\ge 0, y\ge 0$

Also find the corner points.

(xi) Find the centre and radius of the circle $x^2+y^2+12x-10y=0$

(xii) Write equation of tangent to parabola $x^2=16y$ at the point whose abscissa is 8.

(xiii) Show that $10xy+8x-15y-12=0$ represents a pair of straight lines.

(xiv) If $\underline{u}=2\underline{i}-\underline{j}-\underline{k}$ and $\underline{v}=4\underline{i}+2\underline{j}-\underline{k}$, then find the vector $\underline{u}\times\underline{v}$.



## SECTION – C (Marks 40)

**Note: Attempt any FIVE questions. All questions carry equal marks.** **( 5 x 8 = 40 )**

**Q. 3** If $\theta$ is measured in radian, then show that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$.

**Q. 4** Find $\frac{dy}{dx}$ if $y=x\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}$.

**Q. 5** Evaluate $\int\sqrt{a^2+x^2}\ dx$

**Q. 6** Find $h$ such that the point $A(h,1)$, $B(2,7)$ and $C(-6,-7)$ are the vertices of a right triangle.

**Q. 7** Find an equation of a parabola with focus $(-1,0)$ and vertex $(-1,2)$.

**Q. 8** Find the center, foci, eccentricity, vertices and equations of directrices of the hyperbola $\frac{y^2}{16}-\frac{x^2}{9}=1$

**Q. 9** Prove that the midpoint of hypotenuse of a right triangle is equidistant from its vertices.

# MATHEMATICS HSSC–II

## SECTION – A (Marks 20)

Time allowed: 25 Minutes | Version Number | 1 | 8 | 7 | 6 |

**Note:** Section – A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

**Q. 1** Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

1) If $f(x)=\frac{1}{x^2}$ then which of the following is equal to $fof(x)$.

A. $x^4$ B. $x^2$ C. $1$ D. $\frac{1}{x^4}$

2) Find the value of $\lim_{x\to 1}\sqrt{\frac{1-x}{1-\sqrt{x}}}$

A. $\frac{1}{\sqrt{2}}$ B. $0$ C. $\sqrt{2}$ D. $\infty$

3) Find the value of $\frac{d}{dx}\left(\sin^{-1}\sqrt{ax}\right)$

A. $\frac{a}{\sqrt{1-ax}}$ B. $\frac{1}{\sqrt{1-ax}}$ C. $\frac{\sqrt{a}}{\sqrt{1-ax}}$ D. $\frac{\sqrt{a}}{a\sqrt{x(1-ax)}}$

4) If $x=c$ is point of inflection of the function $y=f(x)$ then $f'(c)=0$, $f''(c)=0$ and which of the following is also true for $\varepsilon>0$.

A. $f''(c+\varepsilon)\ f''(c-\varepsilon)<0$ B. $f''(c+\varepsilon)\ f''(c-\varepsilon)>0$

C. $f''(c+\varepsilon)\ f''(c-\varepsilon)=0$ D. $f''(c+\varepsilon)\ f''(c-\varepsilon)=\infty$

5) Evaluate $\frac{d}{dx}\left(ln\left(\frac{1}{e^{ax}}\right)\right)$

A. $a$ B. $-ae^{ax}$ C. $-a$ D. $ae^{ax}$

6) Which of the following is equal to $\frac{d}{dx}(\sinh^{-1}x)$?

A. $\frac{1}{\sqrt{1+x^2}}$ B. $\frac{-1}{\sqrt{1+x^2}}$ C. $\frac{1}{\sqrt{1-x^2}}$ D. $\frac{-1}{\sqrt{1-x^2}}$

7) What is change in volume of the cube if its edges change from 10 cm to 10.1 cm?

A. $300\ cm^3$ B. $3.0\,cm^3$ C. $30\ cm^3$ D. $0.3\,cm^3$

8) Find the value of $\int_0^1 \cosh x\,dx$ ?

A. $\cosh 1$ B. $\sinh 1$ C. $-\sinh 1$ D. $-\cosh 1$

9) Find the value of $\int e^x\left(f(x)+f'(x)\right)dx$ ?

A. $e^x f(x)+c$ B. $e^x+f(x)+c$ C. $e^x f'(x)+c$ D. $xe^x f(x)+c$

10) _____ is the evaluation of $\int\frac{e^{\ln x}}{x}dx$

A. $x+c$ B. $\ln x+c$ C. $\ln(e^{\ln x})+c$ D. $\ln(\ln x)+c$

11) What is equation of the straight line bisecting the 2nd quadrant?

A. $y = x - 1$ B. $y = -x$ C. $y = x + 1$ D. $y = x$

12) Which of the following is the condition that the two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel?

A. $a_1b_2 + b_1a_2 = 0$ B. $a_1b_1 = a_2b_2$ C. $a_1b_2 = b_1a_2$ D. $a_1b_1 + a_2b_2 = 0$

13) What is called the point of intersection of the medians of a triangle?

A. Orthocentre B. In-centre C. Centroid D. Circum centre

14) What is the measure of angle $\angle APB$, if $AB$ is diameter of a circle and P is another point on the circle?

A. $120°$ B. $90°$ C. $150°$ D. $60°$

15) Which of the following points is in the solution set of the linear inequality $2x - 3y - 5 \geq 0$

A. $(2,1)$ B. $(-2,1)$ C. $(2,-1)$ D. $(-2,-1)$

16) _____ is the focus of the parabola $x^2 = -2ay$

A. $\left(0, \frac{-a}{2}\right)$ B. $\left(0, \frac{a}{2}\right)$ C. $\left(\frac{a}{2}, 0\right)$ D. $\left(\frac{-a}{2}, 0\right)$

17) What is centre of the circle if extremities of the diameter are $(-1,1)$ and $(5,-5)$?

A. $(1,-1)$ B. $(2,3)$ C. $(2,-2)$ D. $(-2,2)$

18) Which of the following conics is represented by the equation $x^2 - y^2 = 4$?

A. Circle B. Parabola C. Ellipse D. Hyperbola

19) Which of the following vectors is equal to the vector $\underline{i} . \underline{j} \times \underline{k}$?

A. 0 B. 1 C. $-1$ D. $\underline{i}$

20) What is value of $\lambda$ so that the vectors $2\underline{i} + \lambda\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \lambda\,\underline{k}$ are mutually perpendicular?

A. 1 B. $-1$ C. 2 D. 5

# MATHEMATICS HSSC–II

**Time allowed: 2:35 Hours** **Total Marks Sections B and C: 80**

**NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.**

## SECTION – B (Marks 40)

**Q. 2 Attempt any TEN parts. All parts carry equal marks.** **( 10 x 4 = 40 )**

(i) Evaluate $\lim_{x \to \pi} \frac{\sin x}{\pi - x}$.

(ii) If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, then show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$.

(iii) Evaluate $y = xe^{\sin x}$ w.r.t $x$

(iv) Find dimension of a rectangle of largest area having perimeter 120 centimetres.

(v) Use differential to approximate the value of $\cos 29°$.

(vi) Evaluate $\int \frac{x+2}{\sqrt{x+3}}\,dx$

(vii) Evaluate $\int (\ln x)^2\,dx$ with integration by parts.

(viii) Find the area between x-axis and the curve $y^2 = 4 - x$ in the first quadrant form $x = 0$ to $x = 3$.

(ix) Find the coordinates of the point that divides the join of $A(-6,3)$ and $B(5,-2)$ in the ratio $2:3$.

(x) Find $k$ so that the line joining $A(7,3), B(k,-6)$ and the line joining $C(-4,5)$, $D(-6,4)$ are parallel.

(xi) Find an equation of the line through the point $(2,-9)$ and the intersection of the two lines $2x+5y-8=0$ and $3x-4y-6=0$.

(xii) Find equation of the circle with ends of a diameter at $(-3,2)$ and $(5,-6)$.

(xiii) Write equation of the tangent to the conic $3x^2 = -16y$ at the point whose ordinate is $-3$.

(xiv) Find a vector whose magnitude is 4 and is parallel to vector $2\underline{i} - 3\underline{j} + 6\underline{k}$.



## SECTION – C (Marks 40)

**Note: Attempt any FIVE questions. All questions carry equal marks.** **( 5 x 8 = 40 )**

**Q. 3** Evaluate $\lim_{\theta \to a} \frac{x^m - a^m}{x^n - a^n}$.

**Q. 4** Discuss the function defined by $f(x) = \sin x + \frac{1}{2\sqrt{2}}\cos 2x$ for extreme values in the interval $(0, 2\pi)$.

**Q. 5** Evaluate $\int \sqrt{4 - 5x^2}\,dx$

**Q. 6** Find an equation of the line through $(-4,-6)$ and perpendicular to the line having slope $\frac{-3}{2}$.

**Q. 7** Find an equation of the tangents drawn from point $(0,5)$ to the circle $x^2 + y^2 = 16$.

**Q. 8** Find equation of the common tangent to the conics $x^2 = 80y$ and $x^2 + y^2 = 81$.

**Q. 9** Prove that perpendicular bisectors of the sides of a triangle are concurrent.